MCS CONTENT STANDARDS FOR 2nd GRADE MATHEMATICS

<u>**Fluency Expectations or Examples of Culminating Standards**</u>

- 2.OA.2: ***<u>Fluently</u>*** add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.
- 2.NBT.5: ***<u>Fluently</u>*** add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

<u>The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards</u>:

<u>Significant Changes (ex: change in expecations, new Standards, or removed Standards)</u>
2.NBT.2
2.MD.8

<u>Slight Changes (slight change or clarification in wording)</u>
none

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***<u>fluently</u>***). With respect to student performance <u>and</u> effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***<u>fluently</u>*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

Operations and Algebraic Thinking

Cluster

Represent and solve problems involving addition and subtraction.

Vocabulary: add, addend, sum, subtract, difference, more, less, equal, equation, add to, take from, put together/take apart, compare

Standard

2.OA.1
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Table 1.

(*continued on next page*)

Clarifications

This Standard references addition/subtraction situations that are described in Table 1 (included at the end of this document). In Grade 1, students explored addition/subtraction within 20. In Grade 2, they build on this work to include addition/subtraction within 100. They also build on their work from Grade 1 by exploring "two-step" word problems.

One-step word problems use one operation. Two-step word problems use two operations, which may be the same operation or different operations:

One Step Word Problem One Operation	Two Step Word Problem Two Operations, Same	Two Step Word Problem Two Operations, Different
There are 15 stickers on the page. Brittany put some more stickers on the page. There are now 22 stickers on the page. How many stickers did Brittany put on the page? 15 + □ = 22 or 22 – 15 = □	There are 9 blue marbles and 6 red marbles in the bag. Maria put in 8 more marbles. How many marbles are in the bag now? 9 + 6 = ★ ★ + 8 = □ or 9 + 6 + 8 = □	Carlos has 9 peas on his plate. Carlos ate 5 peas. His mom put 7 more peas on his plate. How many peas are on the plate now? 9 – 5 = ★ ★ + 7 = □ or 9 – 5 + 7 = □

<u>TEACHER NOTES</u>:
(1) Second graders are still developing proficiency with the most difficult subtypes (*Add To/Start Unknown; Take From/Start Unknown; Compare/Bigger Unknown; and Compare/Smaller Unknown*). Two-step problems should **not** involve these particular sub-types.
(2) Most work with two-step problems should involve single-digit addends.
(*Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18)*

Working with physical manipulatives (ex: linking cubes, Base 10 Blocks, ten frames) and drawings (ex: Base 10 drawings, number lines) *before* summarizing their work with equations helps students move through the concrete → pictorial → symbolic progression that promotes long-term understanding.

2.OA.1 (*cont'd*)
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Table 1.

Two Step Word Problem: Two "Easy" Subtypes	**Two Step Word Problem**: One "Easy" and One "Middle Difficulty" Subtype	**Two Step Word Problem**: Two "Middle Difficulty" Subtypes
There were 8 birds in the tree. 4 birds flew away. Then 9 more birds flew into the tree. How many birds are in the tree now?	Maria has 9 apples. Corey has 4 fewer apples than Maria. How many apples do Corey and Maria have in all?	There were 15 kids in the park: 9 boys and some girls. Then some more girls came. Now there are 14 girls in the park. How many more girls came into the park?

(Adapted from *Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18*. Subtype examples and descriptions can be found in Table 1 at the end of this document.)

The goal of Grade 2 is to build on addition/subtraction strategies learned in Grade 1 (ex: making tens, backing down through 10, doubles, doubles +/- 1, place value strategies) by applying them to bigger numbers: By the end of Grade 2, students should no longer depend on counting strategies to add and subtract.

Example of "Making Ten" in Grade 1

$9 + 7 =$

1 6

$10 + 6 = 16$

Example of "Making Tens/Friendly Numbers" in Grade 2

$29 + 47 =$

1 46

$30 + 46 = 76$

Example of "Using Friendly Numbers" in Grade 1

$14 - 9 =$
$14 - 10 = 4$
$4 + 1 = 5$

It's easier for me to subtract 10, so I did. But 10 is 1 too many, so I need to put 1 back to fix it. So, 14 – 9 = 5.

Extending "Using Friendly Numbers" in Grade 2

$44 - 19 =$
$44 - 20 = 24$
$24 + 1 = 25$

It would be easier to subtract 20, so I did. But 20 is 1 too many, so I need to put 1 back to fix it. So, 44 – 19 = 25.

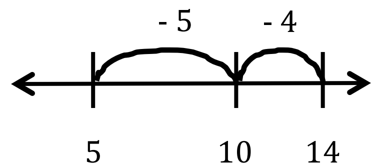
(*continued on next page*)

2.OA.1 (*cont'd*)
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Table 1.

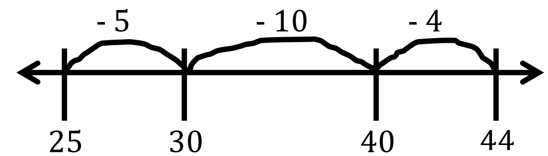
<u>Example of "Backing Down Through 10" in Grade 1</u>

$14 - 9 =$
$14 - 4 = 10$
$10 - 5 = 5$



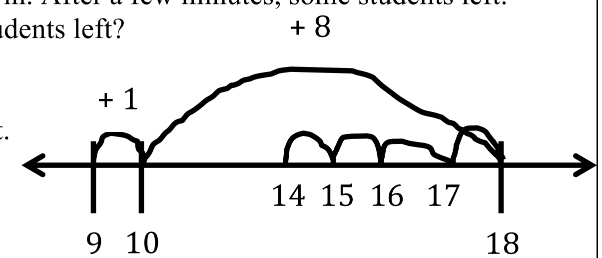
<u>Application of "Backing Down Through 10" in Grade 2</u>

$44 - 19 =$
$44 - 4 = 40$
$40 - 10 = 30$
$30 - 5 = 25$

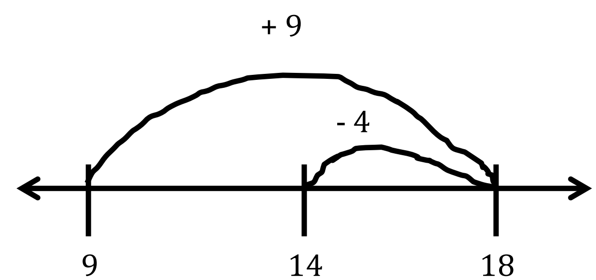


<u>Example</u>:
There are 9 students in the lunchroom. 9 more students come in. After a few minutes, some students left. Then there were 14 students in the lunchroom. How many students left?

<u>Student A</u>:
I started at 9 because that's how many kids there were to start. 9 students came in. I jumped 1 to 10 because I like to work with 10. Then I jumped 8 more for the rest of the kids who came in. Then I jumped back 'til I got to 14 – 1, 2, 3, 4. So 4 students left the lunchroom.



<u>Student B</u>:
There were 9 students in the lunchroom, so I started at 9. 9 more came in -- 9 and 9 is 18 – I know my doubles. Then some students left, and there were only 14 kids. So I jumped back from 18 to 14. 14 is 4 less than 18. So, 4 kids must have left the lunchroom.



Cluster	
Add and subtract within 20.	
Vocabulary: add, addend, sum, subtract, difference, more, less, equal, equation, add to, take from, put together/take apart, compare	
2.OA.2 ***Fluently*** add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers. [2] See standard 1.OA.6 for a list of mental strategies.	"The word *fluent* is used in the Standards to mean 'fast and accurate.' Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., 'adding 0 yields the same number'), and knowing some answers *from the use of strategies*. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students." (*Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18)* **To "know from memory" does not mean to "memorize."** Students have been working on addition within 20 since Grade 1 (**1.OA.1**) and were expected to demonstrate fluency for addition and subtraction within 10 (**1.OA.6**). Students should have numerous opportunities to solve addition/subtraction problems throughout the year so that they internalize number relationships with meaning and context, rather than as stand-alone "facts." Research indicates that teachers can best support students' memory of the sums of two one-digit numbers through varied experiences including making 10, breaking numbers apart, and working on mental strategies. These strategies replace the use of repetitive timed tests in which students try to memorize operations as if there were not any relationships among the various facts. (Fosnot & Dolk, 2001) There has been increasing research within the past several years describing the negative impact that timed tests and drills have on students. The overall consensus is that timed tests and flash card drills are not effective means to help students learn "facts" with long-term success. Students who are strong memorizers may have success with these assessments, but that does not mean that they understand the number relationships. For more information on the negative impact of timed tests and alternative assessment strategies that promote number sense and long-term success, see • Boaler, J. (2014, April). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics*, *20*(8), 469-474. • Kling, G., & Bay-Williams, J. M. (2014, April). Assessing basic fact fluency. *Teaching Children Mathematics*, *20*(8), 488-497. Students who are fluent in number relationships can often reason through 6 + 7 as 6 + 6 + 1, which is 12 + 1 = 13, quickly and efficiently. Students who understand how to use this type of strategy ("doubles + 1") are less likely to make mistakes than students who have tried to memorize facts without any relationships or understanding to support those facts.

Cluster

Work with equal groups of objects to gain foundations for multiplication.

Vocabulary: equal groups, pairs, odd, even, equation, sum, addends

2.OA.3
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

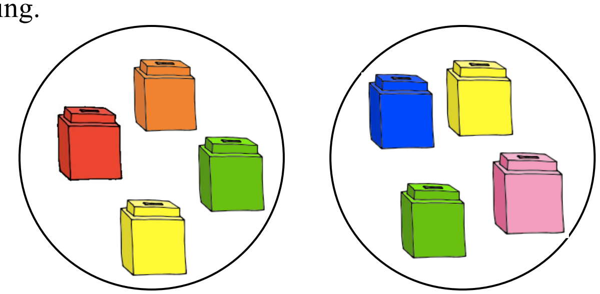
The focus of this Standard is placed on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal (whole number) parts.

The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation *and should not be used as the definition of an even number.* (Van de Walle & Lovin, 2006, p. 292)

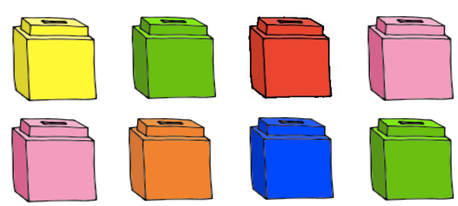
"Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 + 5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays." (Georgia Standards of Excellence Frameworks, GSE Second Grade)

<u>Example</u>: Is 8 an even number? Explain your thinking.

<u>Student A</u>:
I got 8 linking cubes and put them in 2 groups.
There were no leftovers. So, I say 8 is even.



<u>Student B</u>:
I used matching. I got 8 cubes and put them in pairs.
Every cube had a partner, so 8 is an even number.

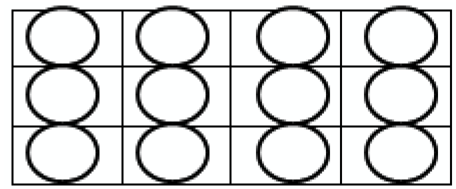


<u>Student C</u>:
You can think of 8 as a double: 4 + 4 = 8.
So, 8 is even.

<table>
<tr>
<td>2.OA.4
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</td>
<td>This Standard lays a foundation for thinking about multiplication as repeated addition in Grade 3 (3.OA.1, 3.MD.7). <u>It is not an expectation for students to learn about multiplication in Grade 2</u>.

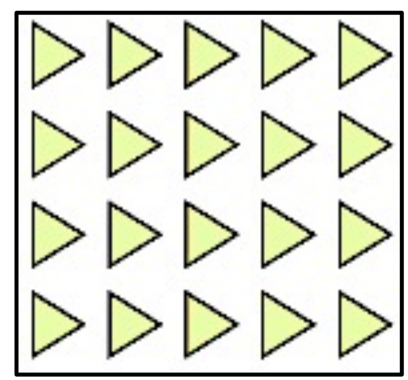
A rectangular array is any arrangement of discrete objects in rows and columns, such as a rectangle of square tiles. Students should explore this concept with concrete objects (e.g., counters, counting bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

<u>Example</u>:
What is the total number of circles? Explain your thinking, and write an equation that best fits your thinking.


<u>Student A</u>:
I see 3 in each column, so I counted by 3s: 3, 6, 9, 12. 12 circles. I would write 3 + 3 + 3 + 3 = 12.

<u>Student B</u>:
I counted how many in each row: 4, 8, 12. So, there are 12 circles. My equation is 4 + 4 + 4 = 12.

<u>Example 2</u>:
Let’s count the triangles are in the box. We could count them one at a time, but is there a faster way we could count them?


<u>Student</u>: There are 5 in each row, so you could count by rows: 5, 10, 15, 20. So, there are 20 triangles.

<u>Example 2 cont’d</u>: Can you write an equation to describe what you did?

<u>Student</u>: (writes) 5 + 5 + 5 + 5 = 20

<u>TEACHER’S NOTE</u>: The triangles in this array are positioned intentionally.
It is important that students see shapes in different orientations. This builds off of students’ work with geometry in Grade 1 (1.G.1) in learning about defining (ex: # of sides) and non-defining (ex: orientation) attributes of shapes.</td>
</tr>
</table>

Number and Operations in Base Ten

Cluster

Understand place value.

Vocabulary: hundreds, tens, ones, skip count, base ten, expanded form, greater than (>), less than (<), equal to (=), digits, compare

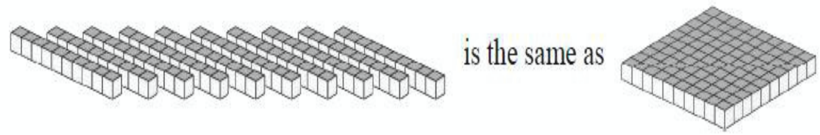
2.NBT.1
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

(*continued on next page*)

In Grade 1, students learned that ten units (or ones) can be grouped together to form a new unit called a "ten." In Grade 2, they build on that understanding to explore grouping 10 tens together to form a new unit called a "hundred." Base 10 Blocks can help students make sense of this relationship in a concrete way.



Understanding the value of the digits is more than telling the number of tens or hundreds. Second Grade students who truly understand the position and place value of the digits are also able to confidently model the number with some type of visual representation. Others who seem like they know, because they can state which number is in the tens place, may not truly know what each digit represents or recognize how the place value amounts are related.

<u>Example of Student *Without* Place Value Understanding</u>:

<u>Teacher</u>: What is this number? 234

<u>Student</u>: Two-hundred thirty-four.

<u>Teacher</u>: Can you model this amount with your Base 10 blocks?

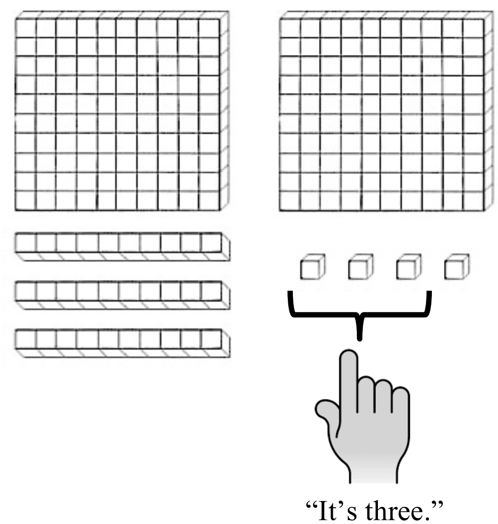
<u>Student:</u> *Uses 2 flats, 3 longs, and 4 units.*

<u>Teacher</u>: <*Pointing to the 4 in 234*>
Can you show me what the 4 is describing in your model?

<u>Student</u>: <*Points to the 4 units*> It's four.

<u>Teacher</u>: <*Pointing to the 3 in 234*>
Can you show me what the 3 is describing in your model?

<u>Student</u>: <*Points to 3 of the 4 units (rather than three longs)*> It's three.

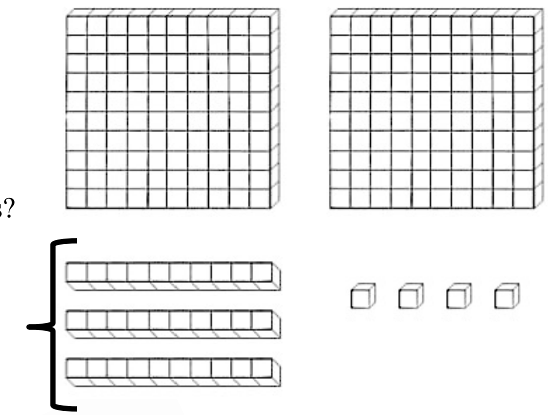


2.NBT.1 (*cont'd*)
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Example of Student *With* Place Value Understanding:



Teacher: What is this number? 234

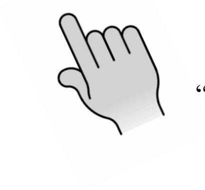
Student: Two-hundred thirty-four.

Teacher: Can you model this amount with your Base 10 blocks?

Student: *Uses 2 flats, 3 longs, and 4 units.*

Teacher: *<Pointing to the 4 in 234>*
Can you show me what the 4 is describing in your model?

Student: *<Points to the 4 units>* Four ones.



"Three tens."

Teacher: *<Pointing to the 3 in 234>*
Can you show me what the 3 is describing in your model?

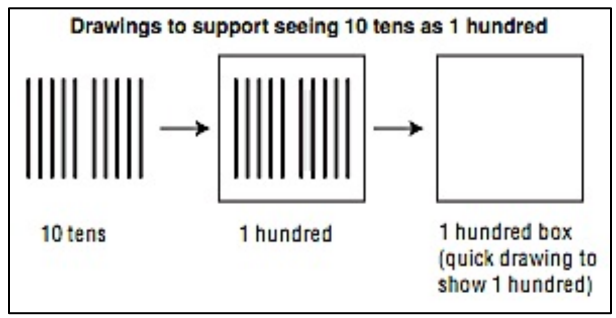
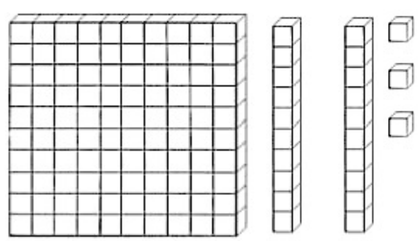
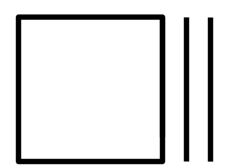
Student: *<Points to the 3 longs>* Three tens.

Second Graders should also have experience with representing hundreds, tens, and ones in different ways.

Example: We are going to work with the number 243. Work with your partner to find as many ways as you can to make 243 with the Base 10 blocks. Write down how many flats, longs, and units you use in this table. I'm going to set the timer for 7 minutes. When it goes off, I want you to stop and look for patterns in the table.

243		
flats	longs	units
2	4	3
1	14	3
0	24	3
2	3	13
2	2	23
2	1	33

Potential Responses Include:
* When the longs go down by 1, the units go up by 10 because you can swap 1 long for 10 units.
* When the flats go down by 1, you can see the longs go up by 10 because you can swap a flat for 10 longs.
* The biggest number in the flats is 2. If you used 3 flats, you'd have 3 hundreds. That's too much for 243.

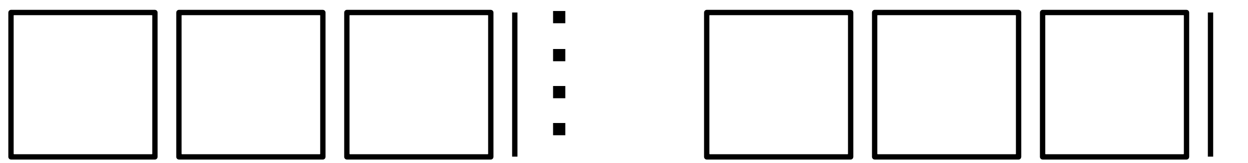
2.NBT.2 Count within 1000; skip-count by 5s starting at any number ending in 5 or 0. Skip-count by 10s and 100s starting at any number.	In Grade 1, students count to 120 (**1.NBT.1**) In Grade 2, students continue counting to 1000 and learn how to "skip-count" (or "count by") 5s, 10s, and 100s. Classroom discussions should bring out number patterns (ex: When skip-counting by fives, the ones digit alternates between 5 and 0.) Counting by 5s and 10s is helpful in working with money & time, also Grade 2 Standards (**2.MD.7**, **2.MD.8**). This Standard lays a foundation for counting by groups and multiplicative thinking, which is a Grade 3 expectation (**3.OA.1**, **3.MD.7**).
2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	It is important for students to understand what the values the digits in numbers represent. For example, we do not traditionally describe the number "523" as "Five two three"; rather, we say, "Five hundred twenty-three." Familiarity with composing and decomposing numbers based on place value helps students use strategies based on place value for addition and subtraction (**2.NBT.5**, **2.NBT.7**). Students should have multiple opportunities to model and talk about numbers with different forms, including physical models, pictures, symbols (digits), and words. ***TEACHER'S NOTE***: When working with three-digit numbers, physically modeling with Base 10 blocks can become unwieldy. It is appropriate to encourage students to draw pictures of Base 10 blocks in lieu of physically representing the numbers with blocks.  (*Progressions for the CCSSM (Draft): Number and Operations in Base Ten, K-5, March 2015, p. 9*) So,  could be quickly drawn as 

<table>
<tr><td>2.NBT.4
Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.</td><td>Second Grade students build on the work of 2.NBT.1 and 2.NBT.3 by examining the amount of hundreds, tens and ones in each number. Base 10 blocks can be very helpful for students learning to compare numbers of this size. Modeling numbers with Base 10 blocks can help students understand that 1 hundred (the smallest three-digit number) is actually more than any amount of tens and ones represented by a two-digit number. When students truly understand this concept, it makes sense that one would compare three-digit numbers by looking at the hundreds place first.

<u>Students should have ample experiences communicating their comparisons in words before using symbols</u>. Students were introduced to the symbols greater than (>), less than (<) and equal to (=) in First Grade and continue to use them in Second Grade with numbers within 1000.

<u>Example</u>:
Compare these two numbers. 314 __ 320.

<u>Student</u>:
I drew pictures of Base 10 blocks to compare the numbers.



<u>Student cont'd</u>: Three hundred fourteen is 3 flats, 1 long, and 3 units. Three hundred twenty would be 3 flats and 2 longs. They have the same number of flats, but 1 long is more than 4 units. So, 314 is less than 320.</td></tr>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Use place value understanding and properties of operations to add and subtract.</td></tr>
<tr><td colspan="2">Vocabulary: compose, decompose place value, digit, more, less, add, sum, subtract, difference</td></tr>
<tr><td>2.NBT.5
<u>Fluently</u> add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

(continued on next page)</td><td>"The word fluent is used in the Standards to mean 'fast and accurate.' Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., 'adding 0 yields the same number'), and knowing some answers <u>from the use of strategies</u>. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that <u>fluency will be a mixture of these kinds of thinking</u> which may differ across students." (Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18)</td></tr>
</table>

2.NBT.5 (*cont'd*)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

This Standard still emphasizes the importance of place value, number relationships, and mathematical reasoning. **Students are not expected to be fluent with the standard algorithms for addition and subtraction until the end of Grade 4 (4.NBT.4)**. However, students can and should have experience in using the traditional algorithm to add and subtract, as it is often efficient.

The word "algorithm" refers to a procedure or a series of steps that when followed will produce a correct solution. Students should be able to explain how they used the traditional algorithm based on understanding of place value and number (**2.NBT.9**). "Explanations" such as "I followed the steps." or "More on the floor? Go next door!" <u>are not mathematical explanations and do not demonstrate deep understanding</u>.

Students should be able to explain their thinking (such as why they chose a particular model or method to solve the problem) and use number sense or estimation to make sure that their answer is reasonable.

<u>Example</u>: 67 + 28 = _____

<u>Student 1</u>:

$$\begin{array}{r} 67 \\ \underline{+\ 28} \\ 80 \\ \underline{15} \\ 95 \end{array}$$

"Partial Sums" Algorithm

<u>Student 2</u>:

67 + 28 = 65 + 30 = 95

65 2

"Making Friendly Numbers"

<u>Student 3</u>:

I changed the numbers to make it easier:

70 + 30 = 100

But then I had to fix it because I changed the numbers. I counted off 3 for 67.

99, 98, 97

And then I counted back 2 more for 28.

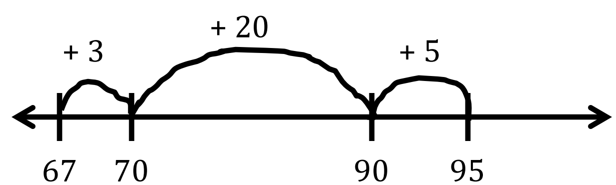
96, 95

So, 67 + 28 = 95

"Compensation" Strategy

<u>Student 4</u>:

I counted up on the number line in my head. 67 and 3 is 70. I did that first because I like round numbers. Then 20 more is 90, and then 5 more is 95. So, 67 + 28 = 95.



"Decomposing Numbers"

(*continued on next page*)

2.NBT.5 (*cont'd*)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Example: $63 - 32 =$ ______

Student 1:

$63 - 30 = 33$

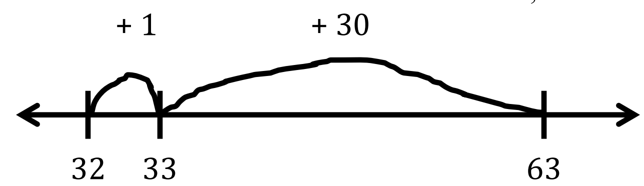
$33 - 2 = 31$

$63 - 32 = 31$

"Subtracting by Place Value"

Student 2:

I counted up on the number line in my head.
32 + 1 is 33. And then 30 more is 63. So, it's 31.



"Relationship between Addition & Subtraction"

Student 3:

I changed the numbers to make it easier: $63 - 33 = 30$. But then I had to fix it because I changed the numbers. I subtracted one too many, so I put one back: $30 + 1 = 31$. So, the answer is 31.

"Compensation" Strategy

TEACHER'S NOTE: Historically when describing the steps of the standard algorithm, phrases such as, "You can't take away a bigger number from a smaller number," are used. This statement should not be used in the classroom because it is inaccurate:

(1) You can subtract a larger amount from a smaller amount. The result is what we call a negative number, often representing the absence of an amount. It is very difficult to get students to "unlearn" this statement when they begin exploring integers (positive and negative numbers) in the sixth grade. It would be better, both for the students' long-term understanding and for consistency of instruction, if students were not presented with such explanations in the elementary grades.

(2) The phrase itself demonstrates an inaccurate interpretation of the mathematics at hand. Consider the problem $423 - 156 =$ __. One might hear, "You can't take away 6 from 3; you can't take away a bigger number from a smaller number!" as the first step in solving this problem. But this problem isn't asking us to "remove 6 from 3"; it's asking us to "remove 156 from 423." We're not trying to take away a bigger number from a smaller number at all. Language such as this strips away the relationship of the digits from the overall value of the number and should be avoided.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.	Second Grade students add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations. <u>Example</u>: 43 + 34 + 57 + 24 = __ <u>Student 1</u> 10 43 + 34 + 57 + 24 40 + 30 + 10 + 50 + 20 = 40, 70, 80, 100 (I added 20 first), 150 Then 154, 158. <u>Student 2</u> 43 + 34 + 57 + 24 First I looked at all the tens: 50 40 + 30 + 50 + 20 50 + 50 = 100 100 + 40 = 140 Then I looked at the ones: 10 3 + 4 + 7 + 4 10 + 4 + 4 = 18 Then I put those together: 140 + 18 = 158 <u>Student 3</u> 43 + 34 + 57 + 24 7 50 = 50 + 34 + 50 + 24 = 100 + 58 = 158

2.NBT.7
Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

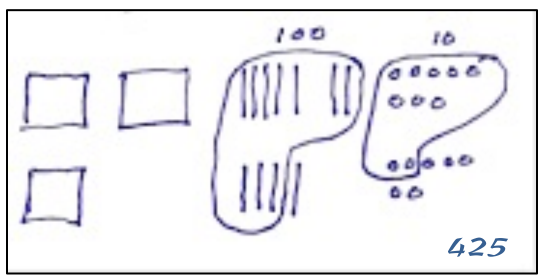
(*continued on next page*)

This Standard emphasizes the importance of models, pictures, number relationships, and mathematical reasoning. **Students are not expected to be fluent with the standard algorithms for addition and subtraction until the end of Grade 4** (**4.NBT.4**). However, students can and should have experience in using the traditional algorithm to add and subtract, as it is often efficient.

The Standards intentionally scaffold addition and subtraction strategies across Grades 1, 2, and 3 The goal is to help students move through developmentally appropriate progression of concrete → pictorial → symbolic stages of understanding, working with larger and larger numbers. <u>Teachers should not skip or hurry through picture and/or modeling strategies when a Standard specifically calls for them</u>.

Grade 1	Grade 2	Grade 3
<u>Add within 100</u> using **concrete models** or **drawings** and **strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction (**1.NBT.4**).	<u>***Fluently*** add and subtract within 100</u> using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (**2.NBT.5**).	
	<u>Add and subtract within 1000</u>, using **concrete models** or **drawings** and **strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method (**2.NBT.7**).	<u>***Fluently*** add and subtract within 1000</u> using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (**3.NBT.2**).

<u>Example</u>: 278 + 147 = ______



Example of using Base 10 Blocks to show combining like units and composing new units

(*Progressions for the CCSSM (Draft): Number and Operations in Base Ten, K-5, March 2015, p. 9*)

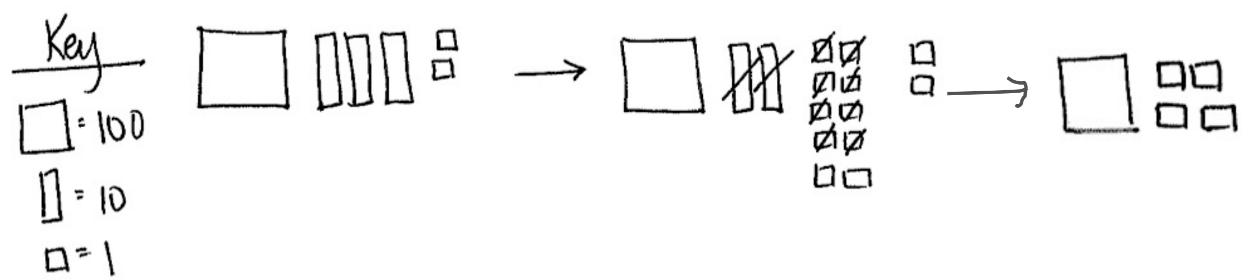
$$\begin{array}{r} 278 \\ +\ 147 \\ \hline 300 \\ 110 \\ 15 \end{array}$$

Young students naturally group the biggest blocks (flats) first, count them, then group the next biggest (longs), and count them, etc. This is <u>mathematically appropriate</u>, as they are grouping by place value: hundreds, tens, and ones.
The writing above shows a way students can record this type of addition strategy.

2.NBT7 (*cont'd*)
Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

(*continued on next page*)

Example: 132 – 28 = _____ 132 – 28 = 104

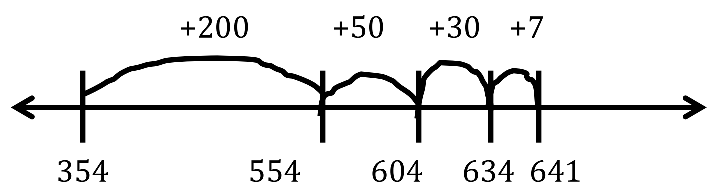


The Base 10 Blocks can become difficult to use when working with larger numbers. A Number Line Model can be a useful strategy for students within this Standard:

* It helps students to use strategies that require composing/decomposing numbers and recognizing place value.
* It gives students a way to organize and record their thinking, which is helpful for them *and* for teachers.
* It allows for a wide range of flexible thinking, depending on how students think about the numbers.
* It allows students to practice mentally adding/subtracting 10 or 100 to/from a number (**2.NBT.8**).
* It is an extension of **2.MD.9**. (**Note**: Some students may jump by tens instead of by multiples of ten. This is normal and developmentally appropriate, as students are still solidifying their mental math skills at this stage.)

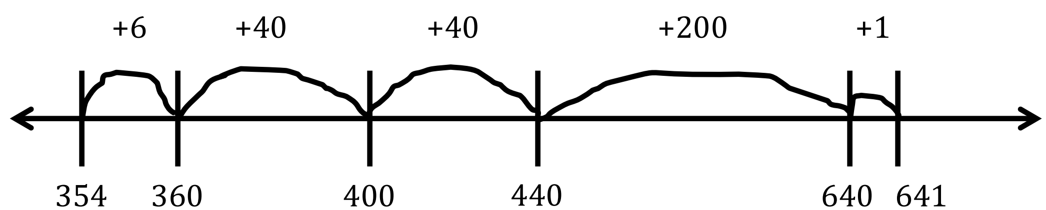
Example: 354 + 287 = __

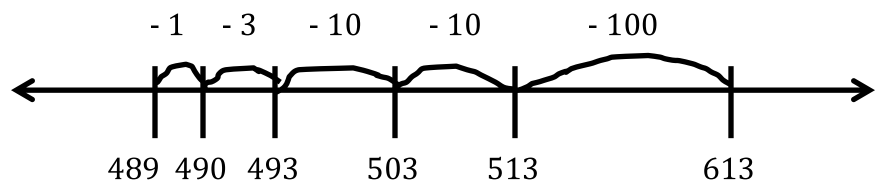
354 + 287 = 641



OR

354 + 287 = 641



2.NBT7 (*cont'd*)	Example: 613 - 124 = __  613 – 124 = 489
2.NBT.8 Mentally add 10 or 100 to a given number 100 – 900, and mentally subtract 10 or 100 from a given number 100 – 900.	Second Grade students mentally add or subtract either 10 or 100 to any number between 100 and 900. Teachers should encourage students to use representations that highlight place value (ex: Base 10 Blocks, the hundreds chart) and prompt in-class discussions of patterns that the students notice. Through these discussions, students should recognize that when they add or subtract 10 or 100, the digit(s) in the tens place and/or the hundreds place change. Opportunities to solve problems in which students cross hundreds are also provided once students have become comfortable adding and subtracting within the same hundred. This Standard focuses only on adding and subtracting 10 or 100. Multiples of 10 or multiples of 100 can be explored; however, the focus of this Standard is to ensure that students are proficient with adding and subtracting 10 and 100 mentally. (See note in **2.NBT.7** on using the Number Line Model to add/subtract.)
2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.[3] [3] Explanations may be supported by drawings or objects.	This Standard is a supporting standard to Standards **2.NBT.5**, **2.NBT.6**, **2.NBT.7**, and **2.NBT.8**. **"Explanations" such as "I followed the steps." or "More on the floor? Go next door!" <u>are not mathematical explanations and do not demonstrate deep understanding</u>.** Students should be able to explain what strategy they chose and how they used it to solve the problem. Rich discussions can include students sharing strategies that they tried but didn't work and then having the class discuss why it might not have worked. The more that students can take ownership of their understanding of mathematical strategies, the more confident they will be. It is important to note that drawings or objects may support explanations, as that is often how students at this age express their thinking/reasoning.

Measurement and Data

Cluster

Measure and estimate lengths in standard units.

Vocabulary: ruler, yardstick, meter stick, measuring tape, length, estimate, longer, shorter, difference, yard, foot, inch, meter, centimeter

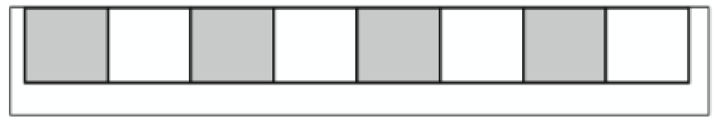
2.MD.1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

(*continued on next page*)

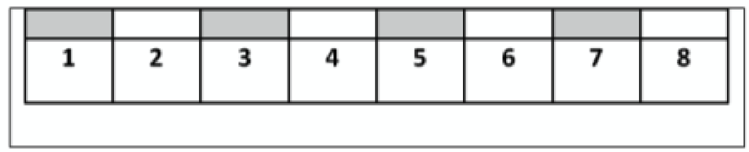
In Grade 1, students measured length by taking small units and iterating (repeating) them, end to end, to cover a distance (**1.MD.2**). In Grade 2, they build on that understanding to use traditional measurement tools and to deepen their understanding of length measurement. Length measurement is a major emphasis in Grade 2.

When Second Grade students are provided with opportunities to create and use a variety of rulers, they can connect their understanding of non-standard units from First Grade to standard units in second grade.

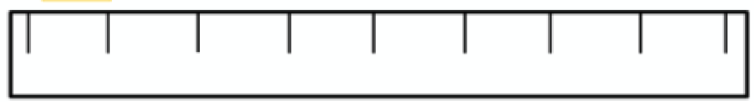
For example, by helping students progress from a "ruler" that is blocked off into colored units (no numbers):

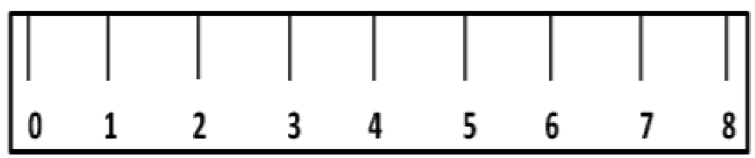


To a "ruler" that has numbers, along with the colored units…



To a "ruler" that has units with and without numbers, students develop the understanding that the numbers on a ruler do not count the individual marks but indicate the spaces (distance) between the marks.





2.MD.1 (*cont'd*) Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes	***TEACHER NOTE***: It is very important to emphasize that when we count measurement units of length, we are counting the *distance traveled* or the spaces; **we are not counting "the hash marks."** This is a critical understanding students need when using such tools as rulers, yardsticks, meter sticks, and measuring tapes. Common Misconceptions for Students with Measurement*: • The numbers on the ruler are counting the hash marks, rather than the spaces between the marks. • When measuring with a ruler, students count the lines instead of the spaces. • Students begin measuring at the end of the ruler instead of at zero. • Students measure at the number 1 instead of at 0 and do not compensate. • Students count intervals on the ruler as the desired interval, regardless of the actual value. • Students lack benchmarks to allow them to estimate and/or self-correct measurements. By the end of Grade 2, students should have a working knowledge of basic measurement relationships: • There are 12 inches in a foot. • There are 3 feet in a yard. • There are 100 centimeters in a meter. *(*Georgia Standards of Excellence Frameworks, GSE Second Grade*)

2.MD.2
Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
	Key concepts within this Standard:
	• Measuring length with different sized units will affect the final measurement.
	  • The smaller the measurement unit, the more units it will take to cover the area.
	  • The larger the measurement unit, the fewer units it will take to cover the area.
	  • Ex: It will take more centimeters than inches to cover the same length because centimeters are smaller than inches. *However*, the actual length of the item being measured does not change. The measurement of that item depends in on the units that were used. This is why it is so important to use units when exploring measurement – i.e., the answer is "24 *inches*," not just "24."
	
	Example:
	Kara and Chris measured a desk. Kara said the desk was 3 feet long. Chris said the desk was 36 inches long. Why do you think they had two different measurements? Did the length of the desk change?
	
	Student: No, it's the same desk. See, inches are a lot smaller than feet, so it takes a lot more inches to cover the length of the desk. Feet are bigger than inches, so you don't need as many feet to cover the same distance.

2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.	Class discussions should facilitate the development of a set of reliable "personal benchmarks" to use as references for measurements within both the metric and customary measurement systems. When students have meaningful references, they are more likely to make sense of measurement relationships. "Reliable" benchmarks refer to using a reference that is not subjective or dependent on the individual. For example, it is often said that an inch is approximately the distance between the first and second knuckles on your index finger. But children's hands are very small (and still growing), so this is not a very reliable reference. Personal benchmarks are not expected to be exact, but they should be reasonable estimates. As such, benchmarks for measurement should always be accompanied by the term "about" or "approximately" because they are not exact measurements. <u>Examples of reliable personal benchmarks for measurement include</u> • About 1 inch – the width of a quarter • About 1 foot – the length of a standard floor tile in a classroom • About 1 centimeter – the length of the edge of a Base 10 unit or the width of a standard pencil eraser • About 1 meter – the distance from the floor to the doorknob on a door <u>***TEACHER NOTE***</u>: The length of a Base 10 unit is about 1 cm. 10 units make a Base 10 long, so a Base 10 long is about 10 centimeters, or 1 decimeter. <u>Key concepts within this Standard</u>: • Students should have multiple experiences with measurement units prior to making estimates; otherwise their estimates will not be reasonable. • Classroom tasks should allow students to predict "about how long" an object will be <u>and then check by actually measuring the object</u>. • Classroom discussions can help students make sense of good estimation strategies. • The classroom lesson "Inch by Inch" in *Math and Literature, Grades 2-3* (Burns & Sheffield, 2004) provides an excellent example of how to do this.
2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	In Grade 1, students compared and ordered three items by length (**1.MD.1**). Grade 2 builds on that experience by asking students to determine not only "Which is longer?" but also "*How much longer* is it?" It is important that students have multiple hands-on opportunities to measure objects with appropriate tools. This allows the teacher to look for common misconceptions (see **2.MD.1**). It is also important that answers include units. An item is not "2 longer" than another; it is "2 inches longer."

Cluster

Relate addition and subtraction to length.

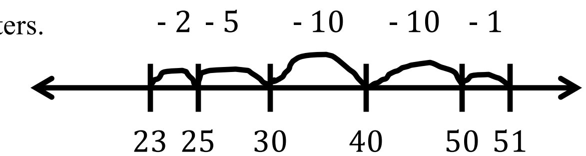
Vocabulary: number line, sum, difference, units, equation

2.MD.5
Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

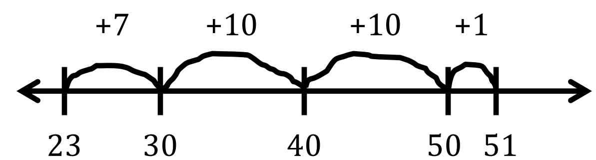
This Standard ties into Second Graders' work with addition/subtraction problems (**2.OA.1**) and the use of a number line diagram to model and solve word problems (**2.MD.6**) by introducing a measurement context.

Example (Compare, Difference Unknown*):
Rick and CJ are on the track team. Rick ran 23 meters. CJ ran 51 meters. Who ran further? By how much?

Student A: CJ ran further. 51 meters is more than 23 meters.
I used a number line. I jumped back from 51 to 23.
Then I counted up how far I jumped: 10, 20, 25, 27, 28.
So, CJ ran 28 more meters than Rick did.



Student B: CJ ran further. I know because 51 is further to the right than 23 is on the number line. First I jumped from 23 to 30 because I like round numbers. Then I kept jumping to 51. Then I added up how far I jumped: 10, 20, 27, 28. CJ ran 28 meters more.



* *See Table 1 at end of document.*

2.MD.6
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.

Important Concepts and "Best Practices" for using Number Line Diagrams

- As we move from left to right on the number line, the number values get larger.
- Jumps should be proportional, but not exact. For example, a jump of 10 should be larger than a jump of 5.
- Marking where we land and how far we jumped helps us keep up with our thinking and communicate our thinking to others.
- Helpful strategies include "friendly jumps" (ex: jumps of 10) or jumping to "friendly numbers" (often multiples of 10).
- To find how far we jumped, we add up "how far we jumped," *not* "the number of jumps." For example, in Student B's number line in **2.MD.5**, he made 4 jumps, but the answer is not "4." The answer is 28 meters – or, *the distance* covered by the jumps that he made.
- Students often begin by jumping by 1s. This is fine initially. Over time, students can be encouraged to use "fewer jumps" or "bigger jumps" in order to build efficiency and mental math skills.

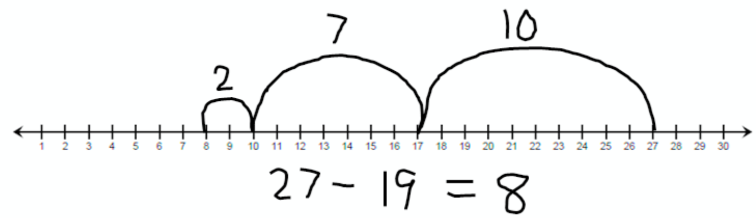
For a helpful teaching article on using Number Line Diagrams, see

Bobis, J. (2007, April). The empty number line: A useful tool or just another procedure? *Teaching Children Mathematics*, 410-413.

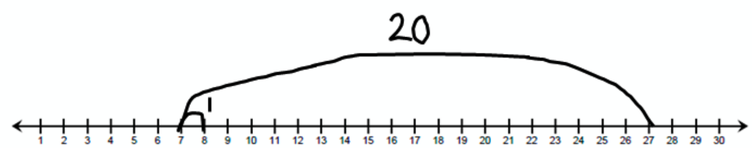
(*continued on next page*)

<table>
<tr><td>2.MD.6 (cont’d)
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram</td><td><u>Example</u> (Take From, Result Unknown):
There were 27 students on the bus. 19 students got off of the bus. How many students are on the bus?

<u>Student A</u> <talking>: I started at 27 because that’s how many kids were on the bus. I jumped back to show the kids getting off of the bus. I jumped 10 first because tens are easier for me. I need to jump 9 more. I jumped 7 first to land on 10. Then I subtracted the last 2 and landed on 8. So, there are 8 students left on the bus.



<u>Student B</u> <talking>: I did it a different way. 19 is really close to 20, and 20 is easier. So, I jumped 20 back to show the kids getting off the bus. But 20 is too many, so I had to jump back one more to make sure I only subtracted 19 – kinda’ like getting a kid back on the bus who wasn’t supposed to get off! I ended up on 8, so there are 8 students left on the bus.


</td></tr>
<tr><td colspan="2">Cluster</td></tr>
<tr><td colspan="2">Work with time with respect to a clock and a calendar, and work with money.</td></tr>
<tr><td colspan="2">Vocabulary: clock, hour hand, minute hand, a.m., p.m., hour, minute, dollar, quarter, dime, nickel, penny, cent(s)</td></tr>
<tr><td>2.MD.7
Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

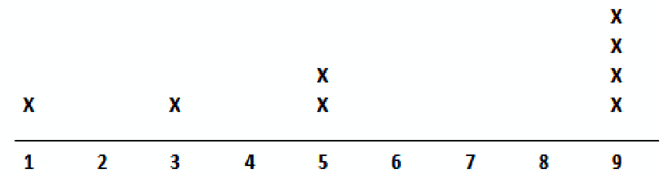
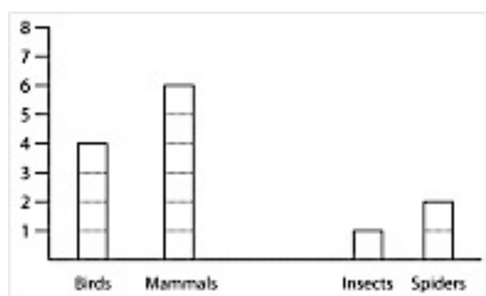
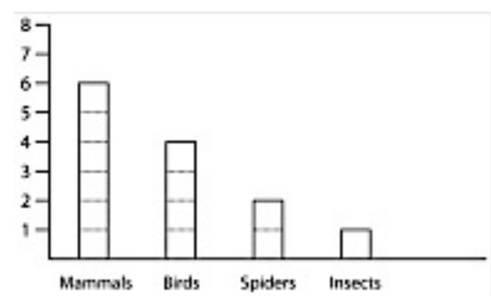
(continued on next page)</td><td>In Grade 1, students learned how to tell time to the nearest hour and half-hour (1.MD.3) This Standard builds on that work and also ties in to students’ work with skip-counting (2.NBT.2).

Students have also partitioned circles into halves and fourths, so they should have at least a visual understanding of what a “half” and “quarter” of an hour (on a traditional clock) would be (1.G.3).</td></tr>
</table>

2.MD.7 (*cont'd*) Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	<u>Scaffolded approaches for helping students with time</u>*: • Begin with a one-handed clock (break the minute hand off of a cheap clock) and use approximate language: "about one o'clock," "a little past three o'clock." • Working with a two-handed clock, discuss the position of the minute hand as the hour hand moves from one number to the next. (Ex: When the hour hand is about halfway between two numbers, where would the minute hand be? When the hour hand is right before a number, about where should the minute hand be?) • Using two clocks (one with two hands, one with only the hour hand), cover the two-handed clock. Throughout the day, ask students to look at the one-handed clock and predict where the minute hand should be. Then uncover the two-handed clock, check, and discuss. • Focus on 5-minute intervals. Guide students beyond predicting, "The minute hand should be at 4" to "It's about 20 minutes after 3:00." A helpful strategy is to focus on the hour first to determine the hour and then the minute hand for more precision. • Work with analog and digital clocks by covering one and then asking students to predict "about what time" should be on that clock, given what time is showing on the other clock. * (Van De Walle, *Elementary and Middle School Mathematics: Teaching Developmentally*, 2007)

2.MD.8
a. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
 	
b. ***<u>Fluently</u>*** use a calendar to answer simple real world problems such as "How many weeks are in a year?" or "James gets a $5 allowance every 2 months, how much money will he have at the end of each year?"
 	
(*continued on next page*)
	Standard **1.MD.5** is new for the 2016-17 academic year. Its purpose is to introduce coins and initial concepts about money to students before they solve problems with money in Grade 2.
	
	Decimal notation is not introduced in the Standards until Grade 4 (**4.NF.6**). Discussions of money in Grade 2 should focus <u>only on whole number amounts</u> with appropriate cent (¢) or dollar ($) notation (ex: 28¢, $4, or a combination, in which describing units with words may be less confusing: 4 dollars and 28 cents).
	
	<u>Research has shown several stumbling blocks we can anticipate as children learn about money</u>:
	• Learning to identify the names/characteristics of the coins
	  • Although some coins are bigger in size than others, students often struggle to distinguish between those that are the same color and relatively close in size (ex: nickel and quarter).
	  • Students may also struggle to feel the difference between "rough edges" (ex: dime and quarter) and "smooth edges" (ex: penny and nickel). **Teachers should be aware that many sets of "play money" have smooth edges <u>for all coins</u>, regardless of their real life counterparts. Please check materials before using them in the classroom.**
	• Learning the "worth" of the coins
	  • Our money is an abstract representation.
	    Ex: A nickel is "worth" 5 cents. A student can't *see* the 5 cents in the nickel; there is nothing there to count, physically – there is only 1 coin, and so the student has to associate that with a quantity of "five cents." Research shows that this type of abstract thinking is challenging for young students but is to be expected as part of the learning progression.

2.MD.8 (*cont'd*) a. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* b. ***Fluently*** use a calendar to answer simple real world problems such as "How many weeks are in a year?" or "James gets a $5 allowance every 2 months, how much money will he have at the end of each year?"	Research has shown several stumbling blocks we can anticipate as children learn about money (*cont'd*): • Our money system is not proportional. Unlike the Base 10 Blocks which are built to represent the concept that "ten ones (ten Base 10 units) are equal to one ten" (one Base 10 long) and "ten tens (ten Base 10 longs) are equal to one hundred (one Base 10 flat), our money system is not designed the same way. **Ex:** A dime is "worth" more than a nickel, but a nickel is physically bigger than a dime. Research shows that it is normal for students to struggle with this reasoning, but it is a convention of our money system that we have to help them accept. Just as students learn that a number (38) can be represented different ways (3 tens and 8 ones; 2 tens and 18 ones) and still remain the same amount (38), students can apply this understanding to money. For example, 25 cents can look like a quarter, two dimes and a nickel, or 25 pennies – All represent 25 cents. This concept of equivalent worth takes time and requires numerous opportunities to create different sets of coins, count sets of coins, and recognize the "purchase power" of coins (a nickel can buy the same things a 5 pennies). "Skip counting" is introduced in the Standards in **2.NBT.2**. The more comfortable students are with skip counting, the easier it will be for them to count up money with different coin values. As teachers provide students with sufficient opportunities to explore coin values (25 cents) and actual coins (2 dimes, 1 nickel), students can apply their knowledge of skip counting (**2.NBT.2**), mental math (**2.NBT.8**), and organizational strategies (**2.NBT.7** - just like we can group place values together, we can group similar coins together to count) to determine the final amount. Example: How many different ways can you make 37¢ using pennies, nickels, dimes, and quarters? Example: How many different ways can you make 12 dollars using $1, $5, and $10 bills? **Note**: MDE has not published guidelines/expectations for new Standard **2.MD.8b** other than the examples given in the Standard. As new information is released, we will make it available to teachers immediately.
Cluster	
Represent and interpret data.	
Vocabulary: data, measure, collect, organize, represent, line plot, picture graph, bar graph, scale, category, how many more, how many less	
2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	This Standard is intended to work with **2.MD.1** by allowing students to organize, represent, and analyze a set of length measurements. The focus is on whole number lengths in Grade 2. A line plot can be seen as a type of number line diagram in that the scale (across the bottom) is represented as a number line. Each observation is represented as an " x " or dot " • " (hence the other name for this type of data display – a "dot plot"). (An example is on the next page.)

2.MD.9 (*cont'd*) Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units	<u>Example</u>: I'm going to walk around the room with the crayon bucket. Each pair will pull two crayons from the bucket and use your rulers to measure how long your crayons are to the nearest centimeter. Then we will work together to build a line plot of what you measured and see what we notice.  <u>Potential Student Observations</u>: * The longest crayons we had were 9 centimeters long. * We had 4 crayons that measured 9 centimeters long. * One crayon was really tiny – about 1 centimeter! Maybe that was just a broken piece. * We didn't have any crayons for certain lengths – like 2 cm, 4 cm. * We measured 8 crayons in all.
2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph. [4] See Table 1.	In Grade 1, students built representations of up to 3 categories of data (**1.MD.4**). These representations may have been bar graphs, picture graphs, tally marks, etc.; the Standards do not specify which to use. In Grade 2, students extend this work, using picture graphs and bar graphs with up to 4 categories of data. They will also answer word problems (**2.OA.1**, Table 1), which require them to interpret these representations. In Grade 3, students will work with <u>scaled</u> picture graphs and bar graphs (**3.MD.3**). It can be helpful for students to see different ways of representing information with bar graphs so that they become comfortable with interpreting them in different formats. For example, the following bar graphs represent the same data in two different ways:   Useful discussions could include, "What is the same and what is different about these two bar graphs?" and "Why might someone make a graph one way or the other?"

Geometry

Cluster

Reason with shapes and their attributes.

Vocabulary: attribute, angles, sides, faces, triangles, quadrilaterals, squares, rectangles, rhombus, rhombuses/rhombi, trapezoids, pentagons, hexagons, circles, cubes, rows, columns, partition, equal shares, halves/half of, thirds/third of, quarters/quarter of, fourths/fourth of, whole

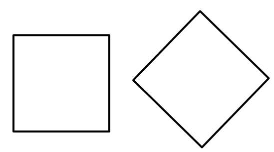
2.G.1
Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

[5] Sizes are compared directly or visually, not compared by measuring.

In Grade 1, students explored defining attributes (ex: number of sides or #number of equal sides) and non-defining attributes (ex: size, orientation) of shapes (**1.G.1**). In Grade 2, students should be able to describe and draw shapes with reasonable accuracy. By the end of Grade 2, students should be able to identify and describe shapes based on their geometric attributes, rather than simply (for example), "It looks like a square."

By "quadrilateral," the Standard does not necessarily mean that students need to learn that term. Students explore the quadrilateral "shape family" in Grade 3 (**3.G.1**). In Grade 2, the Standard uses this term to describe squares, rectangles, parallelograms, trapezoids, and rhombi – quadrilaterals that the students should know.

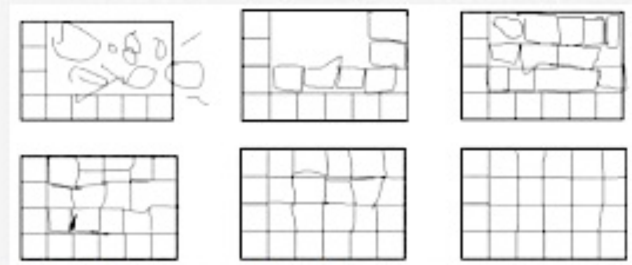
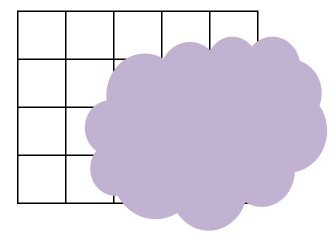
TEACHER NOTE: It is important for students to explore and discuss how squares and rectangles are related. The mathematical attributes of a rectangle *do not include* "having two long sides and two short sides." Those characteristics should not be taught as defining attributes of a rectangle. In its most general terms, a rectangle is a parallelogram that has 4 right angles. (By belonging to the "parallelogram family," we know that a rectangle has two opposite pairs of parallel sides and two opposite pairs of congruent sides.) A square fits all of the characteristics of a rectangle. It is a *special type of rectangle* in that all sides of a square are congruent.



It is also important to note that orientation of a figure does not change the figure itself. Given the shapes above, students often refer to the figure on the left as a square and the figure on the right as a diamond. Both figures are squares; the square on the right has just been rotated.

Unfortunately, many "educational" materials refer to a rhombus, or even a rotated square, as a "diamond." "Diamond" is not a geometric term & should not be used to describe shapes.

TEACHER NOTE: In the U.S., the term "trapezoid" may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as *types of trapezoids*. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (*Progressions for the CCSSM: Geometry*, The Common Core Standards Writing Team, June 2012)

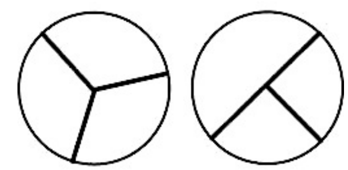
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	This Standard ties into Second Graders' work with arrays (**2.OA.4**) This can be a challenge to students' spatial reasoning. Initially, students may simply draw or place shapes inside a rectangle, without covering the whole region. With time and experience (physical manipulatives such as square tiles may be helpful), they learn how to cover the entire area with rows and columns of same-size square units. **Levels of thinking in spatial structuring**  Levels of thinking portrayed by different students as they attempted to complete a drawing of an array of squares, given one column and row. This was an assessment, not an instructional task. A helpful strategy can be to give students arrays that are partially tiled with squares and then ask them to finish tiling the area with squares. <u>Example</u>: Shelby was studying arrays. She accidentally spilled grape juice on this one. Can you help her figure out how many squares are actually in the array? 
2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (*continued on next page*)	In Grade 1, students split circles and rectangles into halves and fourths (**1.G.2**). In Grade 2, students continue by working with a new partial unit: thirds. Thirds present a new challenge for students because they cannot be obtained easily from the other units they know (whereas fourths can be found by splitting a half in half.) **The focus in Grade 2 is using *names/words* (ex: one half) to describe fractions, not symbols (ex: $\frac{1}{2}$).** (*continued on next page*)

2.G.3 (*cont'd*)
Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
(*continued on next page*)

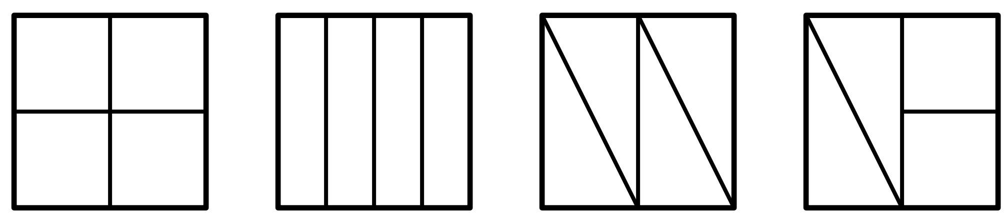
Research shows that "repeated halving" (cutting a whole in half, then cutting those halves in half, etc.) is a powerful strategy for helping students learn how to split one whole into smaller equal parts. The Standards incorporate this research to scaffold students' work with fractions across Grades 1- 4:

Grade	**Expectations**
1st Grade	Halves and fourths (words/names, not symbols/fraction notation)
2nd Grade	Halves, thirds, and fourths (words/names, not symbols/fraction notation)
3rd Grade	Denominators of 2, 4, 8, 3 and 6 (words *and* symbols/fraction notation)
4th Grade	Denominators of 2, 4, 8, 16, 3, 6, 12, 5, 10, and 100 (words *and* symbols/fraction notation)

<u>Important ideas for conceptual understanding of fractions in Grade 2</u>:



- Fractions refer to equal parts, equal shares, or "fair shares."
- The more equal parts the whole is split into, the smaller the parts are.
- The fewer equal parts the whole is split into, the bigger the parts are.
- Fractions reference *a whole*.
- We describe $\frac{3}{4}$ as "three fourths" of a whole, not as "three over four."
- The whole can be described as a fraction, too: two halves, three thirds, four fourths.
- Fractions don't have to be the same shape to represent the same amount. Here are several different ways to split the same square into four equal parts (fourths). Although they look different, each fourth represents the same fraction of the whole as any of the other fourths.



The different representations of fourths above can be challenging for students. It can be a powerful exploration for students to have opportunities to cut out the different "fourths" above and cut them up to see if they do, in fact, cover the same area. (They do.)

Table 1: Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 (1^{st})	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5 One-Step Problem (2^{nd})
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ? (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3 (1^{st})	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3 One-Step Problem (2^{nd})
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ? (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5 or 5 – 3 = ? (1^{st})	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2 (K)
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1^{st})	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? One-Step Problem (1^{st})	(Version with “more”): Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? or ? + 3 = 5 One-Step Problem (2^{nd})
	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5 or 5 – 2 = ? (1^{st})	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ? or 3 + 2 = ? One-Step Problem (2^{nd})	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5 One-Step Problem (1^{st})

K: Problem types to be mastered by the end of the Kindergarten year. **1^{st}**: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year. However, First Grade students should have experiences with all 12 problem types. **2^{nd}**: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous years.

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